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Original Theoretical Research

Probability Grid Method for Fisher-Tippett Law

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Abstract

Introduction. Estimation of the parameters of probability distribution laws using probability grids is widely used in practice, particularly in modern software systems. This approach is actively employed for statistical analysis, where the calculation results are presented as a probability graph. This allows for the assessment of the correspondence between a given data set and a proposed probability model, as well as the identification of outliers. In the context of probabilistic assessment of the loading of machine elements and structures, some authors suggest applying the Fisher–Tippett law. This law is characterized by a distribution function with three parameters and is oriented to the maximum. This provides flexibility in the description of statistical data and enables the estimation of the maximum value in the context of loading. Nevertheless, the existing literature has not sufficiently substantiated the graphical representation of calculation results and the method of parameter estimation, including the use of the probability grid method, which limits the practical application of the Fisher–Tippett law. Therefore, the aim of this study is to justify and develop a methodology for estimating parameters of the Fisher–Tippett law using the probability grid method.

Materials and Methods. The principles and theoretical foundations of constructing probability grids, the preliminary grouping of data, and a ranking method for estimating the empirical distribution were considered as the materials for the study. Analytical dependencies for constructing a probability grid and estimating the parameters of the Fisher–Tippett law were justified. The method of mathematical modeling and comparative analysis were employed. The Matlab 8.6 software package was utilized for modeling. The data were summarized in a tabular format and visualized in the form of graphs.

Results. The method of constructing a probabilistic graph and the method of graphical estimation of the parameters of the Fisher–Tippett law were justified and demonstrated by example. A graph of the empirical distribution function and a probability plot with a description of the locations were presented. A method for constructing a special scale for estimating the shape parameter centered on the origin was proposed. A comparative analysis of parameter estimates obtained using graphical and analytical methods was performed. Estimates of the scale, shape, and shift parameters were compared. The relative error in estimates using the probability grid method was not more than 2%. The indicator for the scale parameter was 1.83%; for the shape parameter was it 0.67%, and for the shift parameter it was 0.45%. Corresponding results of the analytical assessment were 4.4%, 9.33% and 2.13%. In this case, the error was higher, but it did not mean that the analytical method was less accurate.

Discussion and Conclusion. The adequacy of the proposed method of graphical estimation of the parameters of the Fisher–Tippett law by the probabilistic grid method has been demonstrated. This method can be applied, for example, within software packages or user applications. A special scale for graphically estimating the shape parameter can also be used to estimate the shape parameter of the Weibull law. The obtained analytical dependencies, the provisions of the methodology and the graphical materials can be used in the development of the corresponding national standard.

Keywords: probability grid, probability graph, distribution parameter estimation, reliability analysis, Weibull law, Fisher-Tippett law

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Оригинальное теоретическое исследование

Метод вероятностной сетки для закона Фишера – Типпета

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Аннотация

Введение. Оценка параметров вероятностных законов распределения с использованием вероятностных сеток находит широкое применение на практике, особенно в современных программных комплексах. Такой подход активно используется для статистического анализа, где результаты вычислений представляются в виде вероятностного графика, что даёт возможность оценить соответствие набора данных предполагаемой вероятностной модели и выявить выбросы. В контексте вероятностной оценки нагруженности элементов машин и конструкций некоторые авторы предлагают применять закон Фишера – Типпета. Этот закон характеризуется функцией распределения, которая содержит три параметра и ориентирована на максимум, что обеспечивает гибкость в описании статистических данных и позволяет получать оценку максимального значения в контексте нагруженности. Тем не менее, в существующей литературе недостаточно обоснованы графическое представление результатов вычислений и методика оценки параметров, в том числе и с использованием метода вероятностной сетки, что ограничивает практическое применение закона Фишера – Типпета. Таким образом, основная цель данного исследования заключается в обосновании и разработке методики оценки параметров закона Фишера – Типпета с использованием метода вероятностной сетки.

Материалы и методы. В качестве материалов рассматривались принципы и теоретические основы построения вероятностных сеток, предварительная группировка данных и ранговый метод оценки эмпирической функции распределения. Обосновывались аналитические зависимости для построения вероятностной сетки и оценки параметров закона Фишера – Типпета. Использовались метод математического моделирования и сравнительный анализ. Для моделирования задействовали программный комплекс «Матлаб 8.6». Данные обобщали в табличном формате и визуализировали в виде графиков.

Результаты исследования. Обоснована и показана на примере методика построения вероятностного графика и методика графической оценки параметров закона Фишера – Типпета. Представлены график эмпирической функции распределения и вероятностный график с описанием позиций. Предложена методика построения специальной шкалы для оценки параметра формы, ориентированной на точку отсчета в начале координат. Выполнен сравнительный анализ оценок параметров, полученных графическим и аналитическим методами. Сопоставлялись оценки параметров масштаба, формы и сдвига. Относительная погрешность оценок методом вероятностной сетки не превышает 2 %. Показатель для параметра масштаба — 1,83 %; формы — 0,67 %, сдвига — 0,45 %. Соответствующие итоги аналитической оценки: 4,4, 9,3 %3 % и 2,13 %. В данном случае погрешность выше, однако это не значит, что аналитический метод менее точен.

Обсуждение и заключение. Показана адекватность предложенной методики графической оценки параметров закона Фишера – Типпета методом вероятностной сетки. Ее можно применять, например, в программных комплексах или пользовательских приложениях. Специальная шкала для графической оценки параметра формы также подходит для оценки параметра формы закона Вейбулла. Полученные аналитические зависимости, положения методики и графический материал можно использовать при разработке соответствующего национального стандарта.

Ключевые слова: вероятностная сетка, вероятностный график, оценка параметров распределения, анализ надежности, закон Вейбулла, закон Фишера – Типпета

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Introduction. Graphical representation of statistical analysis results in the form of probability graphs is widely used in modern software systems, particularly in the analysis of reliability or survival. This allows for the estimation of distribution law parameters and the identification of outliers[1]. Estimation of parameters using probability grids is used alongside other well-known methods and in some cases may be preferable. Probabilistic graphs are employed in the processing of resource test results[2] and the creation of control maps in quality management systems[3]. The probability grid method enables a visual assessment of the correspondence between a data set and an assumed model for a random variable, as described in the works of M.A. Deryabin [1], S.A. Dobrotin [2], V.L. Shper [3], Ya.I. Bulanov [4], K.S. Ablazova [5], N.P. Velikanova [6], G.Sh. Khazanovich [7] and other modern scientists.

V.E. Kasyanov [8] and A.A. Kotesov [9] propose using one of the forms of generalized distribution of extreme values [10] with a certain type of parameterization, which they propose to call the Fisher–Tippett law, for probabilistic assessment of machine element and structure loading, but differs in that it focuses on maximum values. The Fisher–Tippett law is suitable for estimating reliability indicators in combination with the Weibull law, for example, when applying the load–strength failure model [11].

The graphical representation of the calculation results and the method used to estimate the parameters for the Fisher–Tippett law are not well justified. The scientific literature and regulatory and technical documents do not provide a specific method for estimating these parameters using a probability grid, which limits the practical application of this law. Therefore, the aim of this study is to develop and substantiate a methodology for estimating Fisher–Tippett law using the probability grid method.

Materials and Methods. Estimation of distribution parameters using probability graphs is based on grouping data by intervals and constructing an interval empirical distribution regardless of the assumed theoretical distribution. Therefore, such methods are often called nonparametric or rank-based. A probability grid is constructed for a specific probability distribution law in order to get a linear relationship between variables[4]. Plotting involves linear approximation of an array of empirical points on a probability grid. Therefore, this approach may be considered crude, but it is often used along with others. The probability grid method can be decisive in the case when other methods are untenable. For example, when estimating parameters using the maximum likelihood method, the likelihood function may contain several local maximas. In this case, parameter estimates can be very inaccurate [12].

To justify the probability grid, we need to reduce the probability distribution function to a linear form. The Fisher–Tippett distribution function is defined by the following expression:

$$F(x)=\begin{cases}1-e^{-\left(\frac{c-x}{a}\right)^{b}}, & x\le c,\\ 0, & x>c,\end{cases} \tag{1}$$

where x — value of a random variable; a, b, c — scale, shape, and shift parameters of the distribution, respectively.

We transform distribution function (1) by taking the logarithm of both the left and right sides. Provided that $c > x$, we get:

$$-ln\left(1-F(x)\right)=\left(\frac{c-x}{a}\right)^{b},$$

$$ln\left(-ln\left(1-F(x)\right)\right)=b\,ln(c-x)-b\,ln(a). \tag{2}$$

Obviously, expression (2) is a linear function of the form:

$$y=qx+m, \tag{3}$$

where x — function variable; q and m — constants.

Comparing (2) and (3), we get:

$$\underbrace{ln\left(-ln\left(1-F(x)\right)\right)}_{y}=\underbrace{b\,ln(c-x)-b\,ln(a)}_{qx+m}.$$

Expression (2) differs from a similar sound to the Weibull distribution with three parameters in that only the right side is different:

$$\overbrace{b\,\underline{ln(x-c)}-b\,ln(a)}^{\text{Weibull law}}\quad \overbrace{b\,\underline{ln(c-x)}-b\,ln(a)}^{\text{Fisher}}.$$

[1] GOST R ISO 16269–4–2017. *Statistical methods. Statistical data presentation. Part 4. Detection and treatment of outliers.* Electronic Fund of Legal and Regulatory and Technical Documents (In Russ.) URL: https://docs.cntd.ru/document/1200146680 (accessed: 15.01.2025).
[2] GOST R 50779.27–2017. S*tatistical methods. Weibull distribution. Data analysis.* Electronic Fund of Legal and Regulatory and Technical Documents. (In Russ.) URL: https://docs.cntd.ru/document/1200146523 (accessed: 15.01.2025).
[3] GOST ISO 7870–1–2022. *Statistical methods. Control charts. Part 1. General guidelines.* Electronic Fund of Legal and Regulatory and Technical Documents. (In Russ.) URL: https://docs.cntd.ru/document/1200192703 (accessed: 15.01.2025).
[4] GOST 11.008–75. *Applied statisticas. Graphic methods of data processing. Use of probability papers*. (In Russ.) URL: https://meganorm.ru/Data2/1/4294753/4294753131.pdf (accessed: 15.01.2025).

Therefore, to construct a probability graph of the Fisher–Tippett law, it is advisable to use the basic provisions of GOST 11.008 and GOST 50779.27. According to these standards, statistical data are plotted on a probability grid during graphical analysis, and then the distribution parameters are estimated. Let us note that the probability grid method is implemented both graphoanalytically and completely analytically. Therefore, to eliminate possible ambiguity, we will call the estimation of parameters using the probability grid method graphical, and the estimation by the maximum likelihood method analytical.

The left side of expression (2) allows us to determine the ordinate of the probability scale for estimating the scale parameter. Let us assume that $c - x = a$. By substituting this value in (2), we get:

$$ln\left(-ln\left(1-F\left(x\right)\right)\right)=b\,ln\left(a\right)-b\,ln\left(a\right),$$

$$ln\left(-ln\left(1-F\left(x\right)\right)\right)=0\,,$$

$$e^{ln\left(-ln\left(1-F\left(x\right)\right)\right)}=e^{0}\,,$$

$$-ln\left(1-F\left(x\right)\right)=1\,,$$

$$e^{-ln\left(1-F\left(x\right)\right)}=e^{1}\,,$$

$$\frac{1}{1-F\left(x\right)}=e\,,$$

$$F\left(x\right)=1-\frac{1}{e}\,,$$

$$F\left(x\right)\approx 0.6321\,. \tag{4}$$

Result (4) allows us to conclude that the abscissa of the point approximating the line with zero ordinate will be an estimate of the scale parameter.

The decimal logarithm can be used along the abscissa axis of the probability graph. In this case, dependency (2) will take the form:

$$ln\left(-ln\left(1-F\left(x\right)\right)\right)=\frac{1}{lg\left(e\right)}\left(b\,lg\left(c-x\right)-b\,lg\left(a\right)\right).$$

An important aspect of implementing the probability grid method is the initial processing of statistical data, specifically, obtaining an interval series of variations and estimating the empirical distribution function values. To obtain an empirical distribution function, a ranking method is typically used, which involves estimating the position of a distribution based on ordered data, considering the characteristics of the variation series (mean, median, mode, etc.). Therefore, various dependencies are used to determine the ordinates of points, including expressions for approximate estimation [13]. In this case, the choice will be determined by the amount of empirical data, the expected theoretical distribution, and the type of probability graph. This takes into account the need for an adequate description of the extreme members of the variation series [14].

It should be noted that some previous approaches to estimating the empirical distribution function have been criticized, and this may be the subject of a separate discussion [15].

Results. The inverse function method is used to model a set of random data without a specific physical meaning, distributed according to the Fisher–Tippett law.

The inverse distribution function is obtained analytically from expression (1):

$$F\left(x\right)=1-e^{-\left(\frac{c-x}{a}\right)^{b}}\,,$$

$$-ln\left(1-F\left(x\right)\right)=\left(\frac{c-x}{a}\right)^{b},$$

$$\frac{c-x}{a}=\left(-ln\left(1-F\left(x\right)\right)\right)^{\frac{1}{b}},$$

$$x=c-a\left(-ln\left(1-F\left(x\right)\right)\right)^{\frac{1}{b}},$$

$$F^{-1}\left(x\right)=c-a\sqrt[b]{-ln\left(1-F\left(x\right)\right)}\,. \tag{5}$$

The simulation was conducted using the Matlab 8.6 software package, (Fig. 1), according to the specified parameters *a, b*, and *c.* The initial data used for the modeling are presented in Table 1.

Table 1

Initial data for modeling

Parameters of the Fisher–Tippett law			Number of values
a	*b*	*c*	*n*
100.00	3.00	250.00	100

```
fisher_tippet.m*
'@Исходные данные:';
a = 100; '/параметр масштаба';
b = 3;   '/параметр формы';
c = 250; '/параметр сдвига';

'@Генерация массива случайных числел в интервале [0;1] с помощью команды rand():';
gamma = rand(100,1);
'@Вычисление значений обратной фукнкции закона Фишера-Типпета и получение случайной выборки в виде массива data:';
data = c-a*((-log(1-gamma)).^(1/b));

'@Оценка параметров распределения по ранее полученной случайной выборке методом максимального правдоподобия:';
'@Функция плотности распределения закона:';
custompdf = @(x,a,b,c) (c>x).*(b/a).*(((c-x)/a).^(b-1)).*exp(-((c-x)/a).^b);
'@Точность и количество итераций:';
opt = statset('MaxIter',1e5,'MaxFunEvals',1e5,'FunValCheck','off');
'@Оценка параметров распределения с помощью команды mle()';
params = mle(data,'pdf',custompdf,'start',[100 3 250],'Options',opt,'UpperBound',[0 0 -Inf],'UpperBound',[max(data) Inf Inf]);
```

Fig. 1. Modeling a set of random data in Matlab 8.6

The simulation results in the form of a set of random data x_i are presented in Table 2.

Table 2

A set of random data with no definite physical meaning

No.	x_i									
1	201.98	222.87	182.26	183.98	133.30	114.41	204.15	157.16	169.63	217.17
2	124.97	100.63	138.10	112.03	185.71	160.66	169.88	123.02	192.45	179.76
3	143.79	97.90	118.26	208.58	152.80	95.93	179.54	214.92	155.05	132.63
4	140.21	199.05	140.76	179.14	200.77	189.65	178.47	117.03	152.32	174.79
5	148.32	164.27	169.47	153.61	160.16	200.97	201.86	198.03	187.74	205.69
6	160.11	147.75	109.29	188.97	127.93	179.33	153.42	128.49	159.80	160.55
7	176.62	180.02	183.43	149.66	113.64	170.37	180.74	132.75	84.58	172.97
8	147.27	138.01	158.67	133.01	161.65	168.27	194.75	114.29	162.36	139.61
9	199.99	156.53	104.26	161.36	181.23	178.00	241.30	197.14	144.12	159.39
10	195.72	167.66	182.20	148.29	148.13	144.22	180.65	161.10	169.07	132.26

An analytical assessment of the scale, shape, and shift parameters has been conducted. The estimates are indicated respectively — a', b', c' (Table 3).

Table 3

Results of the analytical evaluation of the parameters

Estimates of the parameters of the Fisher–Tippett law		
a'	b'	c'
104.40	3.28	255.32

The Fisher–Tippett law, unlike the Weibull law, has a restriction on the right and sets the maximum value for a random variable. Therefore, to obtain a series of variations, it is necessary to order the values in the dataset (sample) from maximum to minimum.

If the sample size is $n \leq 30$, then it is not advisable to group the data by intervals. In such cases, each variant should be assigned a rank of j, and an approximation for the median position of these ranks can be used to estimate the values of the empirical distribution function [16]:

$$F(x_i) = \frac{j-0,3}{n+0,4}(j = 1,2\ldots n), \tag{6}$$

where x_i — value of the sample variants ordered from maximum to minimum, corresponding to the j-th rank; j — ordinal number of the rank; n — sample size.

Otherwise, for $n > 30$ it is necessary to group the data by intervals in accordance with the absolute sample size. At the same time, it is recommended to take the number of interval k in the range of $7 \leq k \leq 40$ depending on sample size n. To group the data, it is necessary to determine the boundaries of the interval by selecting the values $X' \leq x_{\min}$ and $X'' \geq x_{\max}$, and divide the resulting interval $[X'; X'']$ into the intervals of equal length h:

$$h = \frac{X'' - X'}{k}. \tag{7}$$

Then, an interval variation series should be obtained by determining the number of sample values n_i, that fall within each interval. Each interval is described by abscissa X_i, which defines the position of the ordered data distribution.
For the middle position, the empirical distribution function is estimated using the expression:

$$F(X_i) = \sum_{i=1}^{k} \frac{n_i}{n+1}(i = 1,2\ldots k), \tag{8}$$

where X_i — middle of the i-th interval; n_i — number of sample members that fell into the i-th interval; k — number of intervals; n — sample size.

As an example, the values of the empirical distribution function were grouped and calculated (Fig. 2) for the data set from Table 3.

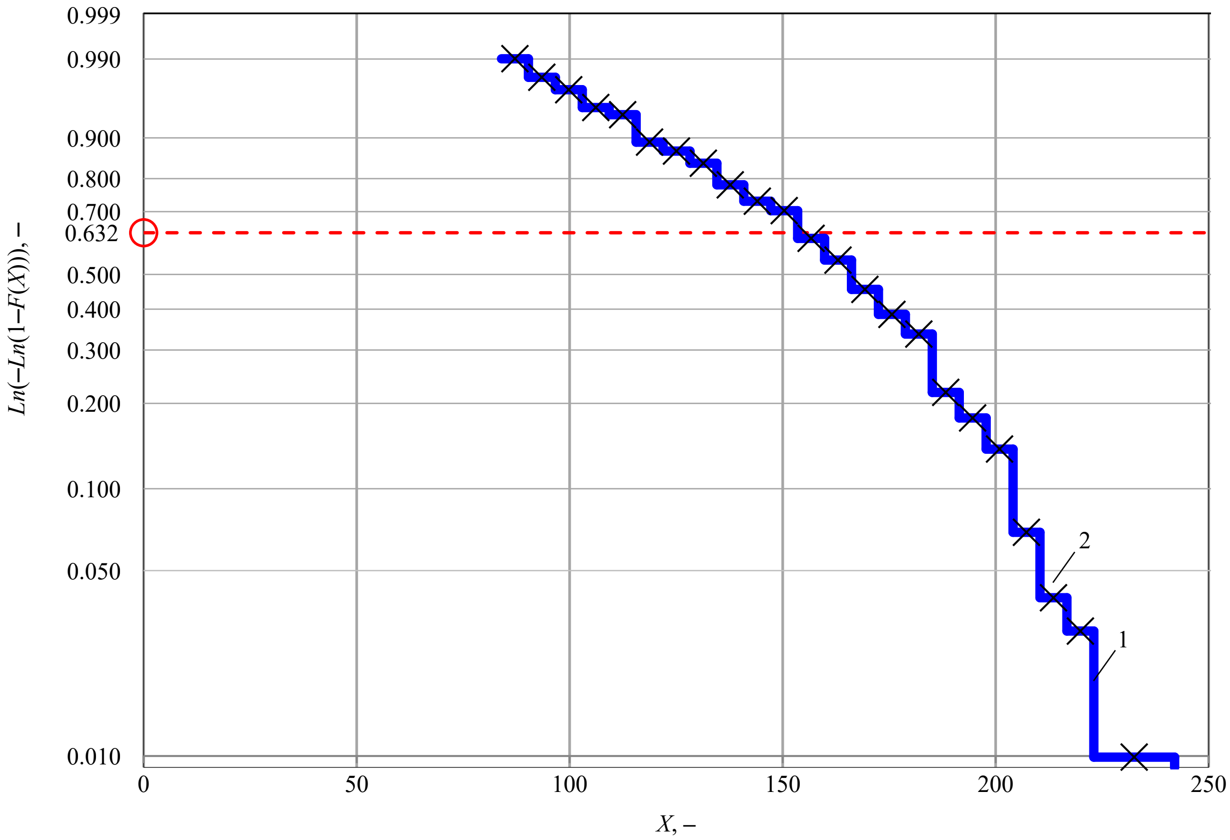

Fig. 2. Empirical distribution function: 1 — function; 2 — middle of the interval

Figure 2 shows the probability value along the ordinate axis; and the values of the dataset (sample) without a specific physical meaning are shown along the abscissa axis.

For data grouping, we assume $k = 25$, $X' = 84$, $X'' = 242$ and determined value $h = 6.32$. One sample value falls into the first three intervals, so they are combined. The total number of intervals — $k = 23$. Table 4 provides the calculation results.

Table 4

Calculation results

i	Rank interval		n_i	X_i	$F(X_i)$	$F(X_i)+F(X_{i+1})$	$Lg(X_i)$	$Ln(-Ln(1-(F(X_i)+F(X_{i+1}))))$	$C'-X_i$	$Lg(C'-X_i)$
	beginning	end								
1	2	3	4	5	6	7	8	9	10	11
1*	**223.04***	**242.00***	1	232.52	0.0099	0.0099	2.3665	–4.6101	22.68	1.3555
2	216.72	223.04	2	219.88	0.0198	0.0297	2.3422	–3.5015	29.00	1.4623
3	210.40	216.72	1	213.56	0.0099	0.0396	2.3295	–3.2087	35.32	1.5480
4	204.08	210.40	3	207.24	0.0297	0.0693	2.3165	–2.6335	41.64	1.6195
5	197.76	204.08	7	200.92	0.0693	0.1386	2.3030	–1.9024	47.96	1.6808
6	191.44	197.76	4	194.60	0.0396	0.1782	2.2891	–1.6282	54.28	1.7346
7	185.12	191.44	4	188.28	0.0396	0.2178	2.2748	–1.4038	60.60	1.7824
8	178.80	185.12	12	181.96	0.1188	0.3366	2.2600	–0.8906	66.92	1.8255
9	172.48	178.80	5	175.64	0.0495	0.3861	2.2446	–0.7175	73.24	1.8647
10	166.16	172.48	7	169.32	0.0693	0.4554	2.2287	–0.4979	79.56	1.9007
11	159.84	166.16	9	163.00	0.0891	0.5446	2.2122	–0.2402	85.88	1.9339
12	153.52	159.84	7	156.68	0.0693	0.6139	2.1950	–0.0497	92.20	1.9647
13	147.20	153.52	9	150.36	0.0891	0.7030	2.1771	0.1939	98.52	1.9935
14	140.88	147.20	3	144.04	0.0297	0.7327	2.1585	0.2771	104.84	2.0205
15	134.56	140.88	5	137.72	0.0495	0.7822	2.1390	0.4214	111.16	2.0459
16	128.24	134.56	6	131.40	0.0594	0.8416	2.1186	0.6111	117.48	2.0699
17	121.92	128.24	3	125.08	0.0297	0.8713	2.0972	0.7179	123.80	2.0927
18	115.60	121.92	2	118.76	0.0198	0.8911	2.0747	0.7963	130.12	2.1143
19	109.28	115.60	5	112.44	0.0495	0.9406	2.0509	1.0379	136.44	2.1349
20	102.96	109.28	1	106.12	0.0099	0.9505	2.0258	1.1005	142.76	2.1546
21	96.64	102.96	2	99.80	0.0198	0.9703	1.9991	1.2575	149.08	2.1734
22	90.32	96.64	1	93.48	0.0099	0.9802	1.9707	–4.6101	155.40	2.1914
23	84.00	90.32	1	87.16	0.0099	0.9901	1.9403	–3.5015	161.72	2.2088

where * — Correction when combining intervals 1–3 into one interval [223,04; 242,00]

On the x-axis of the probability graph, we use a scale with decimal logarithms. The calculation results from columns 8 and 9 of Table 4 determine the coordinates of the points for plotting $\{Lg(X_i); Ln(-Ln(1-(F(X_i)+F(X_{i+1}))))\}$.

At the next stage, the shift parameter is evaluated. To do this, a smooth curve (rather than a straight line) (Fig. 3, pos. 2) should be drawn through the array of points (Fig. 3, pos. 1).

At the point where the straight line intersects the zero ordinate (Fig. 3, pos. 7), graphical estimation of scale parameter A' is made.

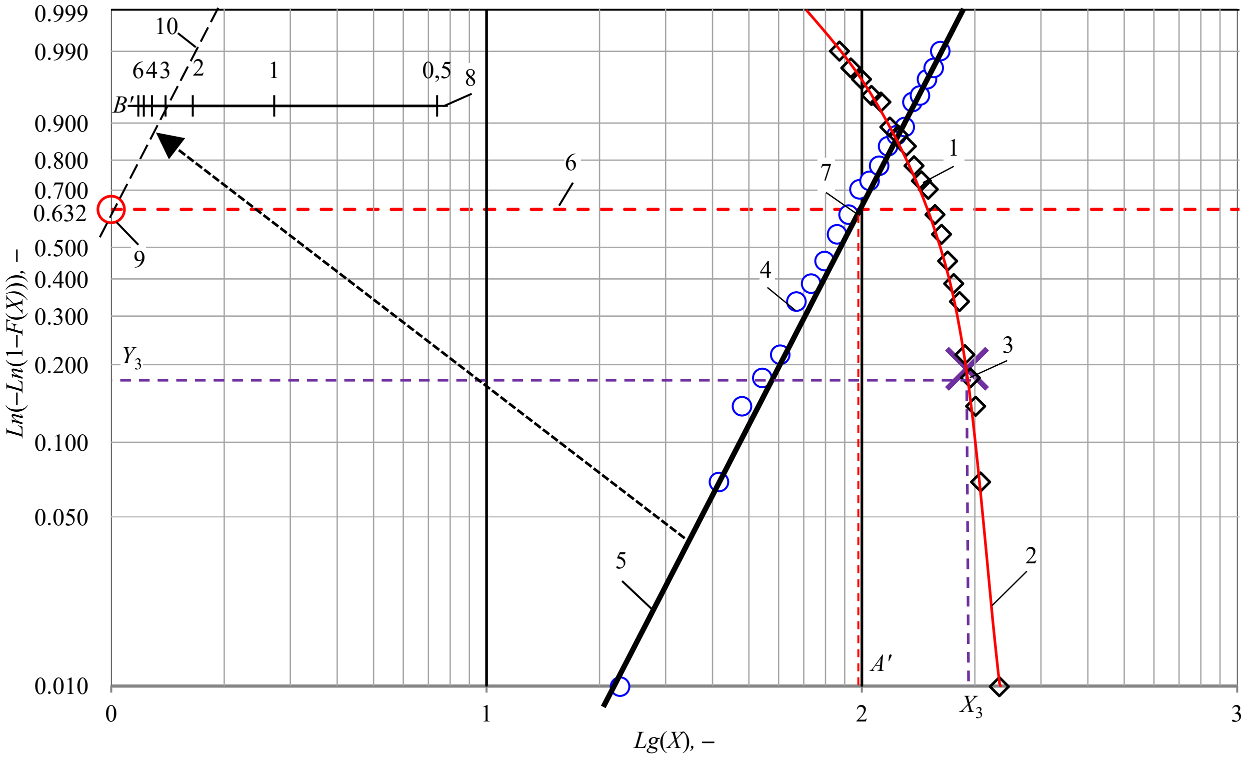

Fig. 3. Graphical estimation of the Fisher–Tippett law parameters: 1 — points with coordinates $\{Lg(X_i); Ln(-Ln(1-(F(X_i)+F(X_{i+1}))))\}$; 2 — line for estimating abscissa X_3 along ordinate Y_3; 3 — point with coordinates $\{Y_3; X_3\}$; 4 — points with coordinates $\{Lg(C'-X_i); Ln(-Ln(1-(F(X_i)+F(X_{i+1}))))\}$; 5 — straight line approximating points 4; 6 — line for estimating the scale parameter; 7 — intersection point of lines 5 and 6, advising the estimation of scale parameter A'; 8 — scale for estimating shape parameter B'; 9 — point with coordinates $\{0; 0\}$; 10 — straight line drawn through point 9 parallel to straight line 5, to evaluate shape parameter B' on scale 8

Figure 3 shows the probability value along the ordinate axis, and the abscissa axis shows the values of a data set (sample) without a specific physical meaning.

The coordinates of the points of the extreme members of the variation series are denoted by $\{X_1; Y_1\}$ and $\{X_2; Y_2\}$, and Y_3 coordinate is estimated:

$$Y_3 = \frac{Y_1 + Y_2}{2}. \tag{9}$$

Using ordinate Y_3 on the previously indicated curve, abscissa X_3 can be determined (Fig. 3, pos. 3). Then shift parameter C' is estimated:

$$C' = \frac{X_1 \cdot X_2 - X_3^2}{X_1 + X_2 - 2X_3}. \tag{10}$$

In the example presented, the extreme terms of the variation series are the midpoints of intervals $i = 1$ and $i = 23$ with coordinates $\{Lg(X_I); Ln(-Ln(1-(F(X_I))))\}$ and $\{Lg(X_{23}); Ln(-Ln(1-(F(X_{22})+F(X_{23}))))\}$. Accordingly, $Y_1 = Ln(-Ln(1-(F(X_I)))$, $Y_2 = Ln(-Ln(1-(F(X_{22})+F(X_{23})))$, $X_1 = Lg(X_1)$; $X_2 = Lg(X_{23})$. As a result, a graphical estimate of shift parameter $C' = 248.88$. We use it to adjust the abscissa of all points, determining values $(C'-X_i)$, and plot the points with the corresponding coordinates on the graph (Fig. 3, pos. 4). As you can see, after the correction, the points lined up "more evenly", which allows us to draw a straight line through them (Fig. 3, pos. 5).

The estimate of the shape parameter corresponds to the angle of inclination of the approximating straight line (Fig. 3, pos. 5) to the abscissa axis. To graphically evaluate the parameter, you can use the coordinates of the points or a special scale (if available). When estimating the shape parameter by coordinates, it is necessary to express the values along the abscissa axis on the scale of the natural logarithm, i.e. use the value $Ln(X)$ instead of $Lg(X)$.

The considered example shows a scale for graphical evaluation of shape parameter B' (Fig. 3, pos. 8). To construct the scale, the coordinates of points $\{Lg(X); Ln(Y)\}$ were calculated based on the set values of the shape parameter (Table 5). The scale is oriented to the reference point with coordinates $\{0; 0\}$ (Fig. 3, pos. 9). To estimate the shape parameter, it is necessary to draw a straight line parallel to the approximating line through the reference point (Fig. 3, pos. 10).

Table 5

Building a scale for graphical evaluation of the shape parameter

B´	0.5000	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000
Ln(*Y*)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Ln(*X*)	2.0000	1.0000	0.5000	0.3333	0.2500	0.2000	0.1667
Lg(*X*)	0.8686	0.4343	0.2171	0.1448	0.1086	0.0869	0.0724

As a result of data processing, graphical estimates of the parameters of the Fisher–Tippett law were obtained (Table 6).

Table 6

Results of graphical parameter estimation

Estimates of the parameters of the Fisher–Tippett law		
A´	*B´*	*C´*
98.17	2.98	248.87

After evaluating the parameters, we need to perform a check using inverse function (5) with the specified probability values:

$$F^{-1}(x) = C' - A'\left(-\ln\left(1 - F(x)\right)\right)^{\frac{1}{B'}}. \quad (11)$$

By calculating the values of inverse distribution function (11) and connecting the resulting points on the graph, you can visually assess the quality of the model. As you can see, the graph of the inverse function (Fig. 4) smoothly describes the array of initial points (Fig. 4, pos. 1 and 2). This suggests that the model is able to accurately describe the data, and the parameter estimation has been performed correctly.

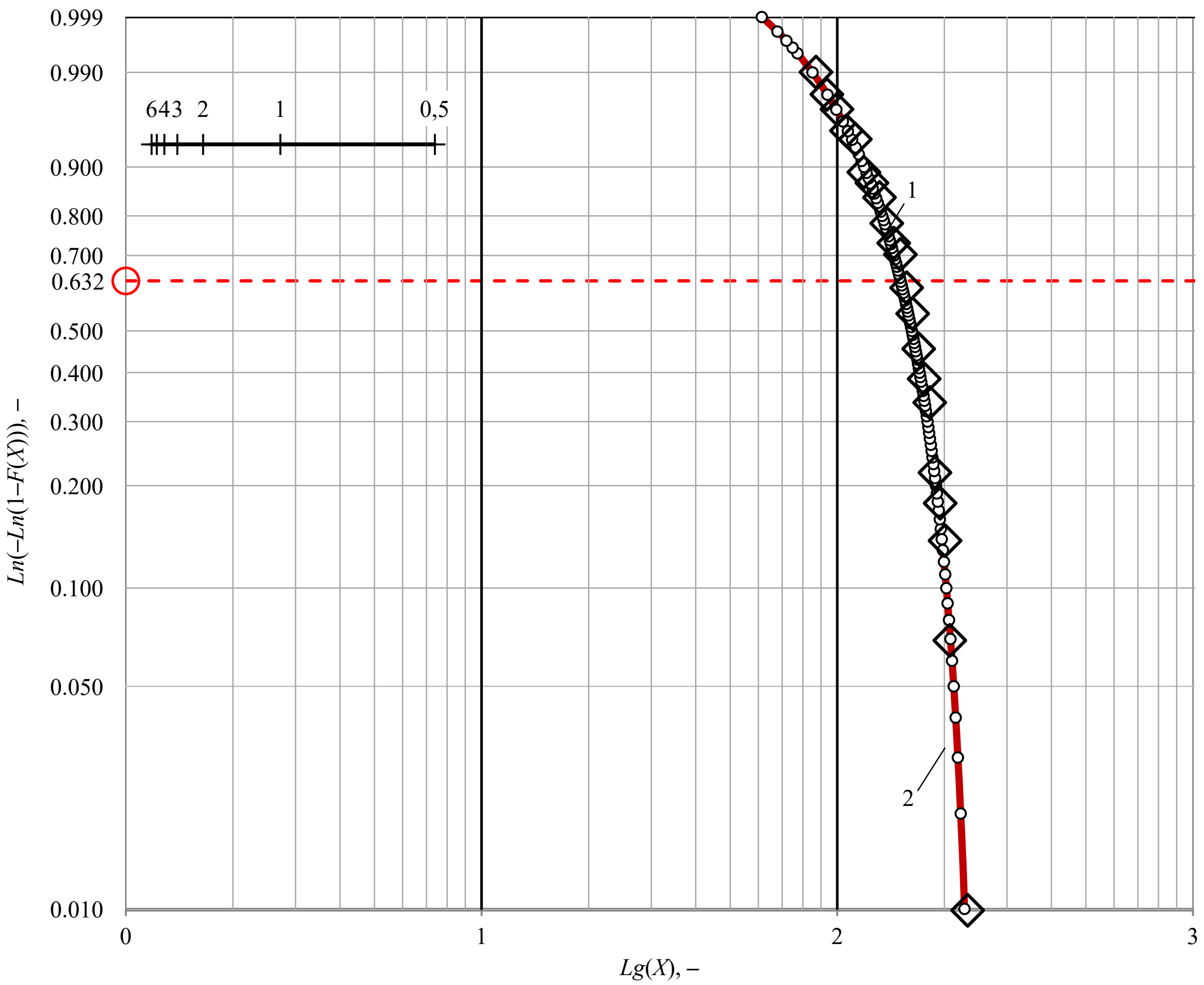

Fig. 4. Checking the model after graphical evaluation of the parameters
1 — starting points with coordinates $\{Lg(X_i); Ln(-Ln(1-(F(X_i)+F(X_{i+1}))))\}$;
2 — graph of inverse distribution function $F^{-1}(x)$ with parameters *A´*, *B´*, *C´*

Figure 4 shows the probability value along the ordinate axis, and the abscissa axis shows the values of the data set (sample) without a specific physical meaning.

The results of the verification calculations are presented in Table 7.

Table 7

The results of the model's testing

$F(x)$	$F^{-1}(x)$	$Lg(F^{-1}(x))$	$Ln(–Ln(1–(F(x))$
0.0010	239.1548	2.3787	–6.9073
0.0050	232.2033	2.3576	–5.2958
0.0100	227.8309	2.3068	–4.6001
0.0500	212.5564	2.2775	–2.9702
0.1000	202.6589	2.2537	–2.2504
0.2000	189.4586	2.2317	–1.4999
0.3000	179.3564	2.2096	–1.0309
0.4000	170.4725	2.1862	–0.6717
0.5000	162.0373	2.1596	–0.3665
0.6000	153.5320	2.1263	–0.0874
0.7000	144.4053	2.0758	0.1856
0.8000	133.7435	2.0299	0.4759
0.9000	119.0766	1.9303	0.8340
0.9900	85.1730	1.8882	1.5272
0.9990	61.3715	1.7880	1.9326

As it can be seen, the graphical and analytical estimates of the parameters are close to the parameters set during the modeling of the dataset (*a*, *b*, *c*).

It is not entirely correct to compare the estimates obtained with respect to the specified parameters, however, such a comparison is justified if the specified parameters are taken as the true parameters of the general population, and the set of random data x_i is considered a representative sample. A comparative analysis of graphical and analytical estimates is presented in Table 8.

Table 8

Comparison of graphical and analytical estimates of parameters

Indicator	Scale parameter	Value	δ, %	Shape parameter	Value	δ, %	Shift parameter	Value	δ, %
Preset parameters	*a*	100.00	–	*b*	3.00	–	*c*	250.00	–
Analytical estimation of parameters	*a′*	104.40	4.40	*b′*	3.28	9.33	*c′*	255.32	2.13
Graphical estimation of parameters	*A′*	98.17	1.83	*B′*	2.98	0.67	*C′*	248.87	0.45

Comparative analysis has shown that the relative error of graphical estimates does not exceed 2% ($\delta < 2\%$). The error of the analytical estimates in this example turned out to be higher, but this does not mean that the analytical method is less accurate.

Discussion and Conclusion. The presented probability grid method for the Fisher–Tippett law is adequate and suitable for practical application. For example, it can be used in software packages or when creating custom applications for graphical representation of statistical analysis results. It opens up the possibility to perform model fitting together with other known methods, even if they are untenable. The proposed method of constructing a scale for graphically estimating the shape parameter can be used to evaluate the shape parameter of the Weibull's law. The obtained analytical dependencies, the provisions of the methodology and the graphic material can be useful in the development of an appropriate national standard.

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